A Study of the Emittances of P1 during Proton Injections for Store 2070

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The following quantities for proton bunch 1 were extracted from SDA Viewer for the 36 Sets of the Inject Proton Case in store 2070 (12/12/02).

T:SBDPSS – SBD rms bunch length *(Is this really the rms value?)*
T:SBDPPS – SBD Δp/p
T:FWHPSG - sigma of E11 horizontal FW (1st and 2nd passes)
T:FWEPSG – sigma of E17 horizontal FW (1st and 2nd passes)
T:FWVPSG – sigma of E11 vertical FW (1st and 2nd passes)
T :WHEP00 – FW horizontal emittance
T :WVEP00 – FW vertical emittance
T :WEEP00 – FW Δp/p

All SDA data appears to be good except the vertical FW emittance of Set 33 reports an error.

Recalculation of Δp/p:

For a bunch matched to a stationary RF bucket generated by a sinusoidal RF waveform, the equations of motion are

$$\frac{d}{dt}\left(\frac{\delta E}{h\omega_0}\right) = \frac{e\hat{V}\sin\phi}{2\pi h}, \qquad \frac{d\phi}{dt} = \frac{-h^2\omega_0^2\eta}{E_0}\left(\frac{\delta E}{h\omega_0}\right)$$

where h = harmonic number, ω_0 = angular revolution frequency, δE = energy deviation from synchronous particle, $e\hat{V}$ = peak RF voltage, ϕ = phase with respect to RF (0 – 2π), E_0 = central energy, and η = slip factor. Assuming conservation of charge in 2-dimensional phase space, $\vec{\nabla} \bullet \rho\vec{V} = 0$, gives a differential equation for the phase space density:

$$\frac{h^2\omega_0^2|\eta|}{E_0}\frac{\delta E}{h\omega_0}\frac{\partial\rho}{\partial\phi} + \frac{e\hat{V}\sin\phi}{2\pi h}\frac{\partial\rho}{\partial\left(\frac{\delta E}{h\omega_0}\right)} = 0.$$

If the distribution is separable, $\rho = \rho_\phi(\phi)\rho_{\delta E}\left(\frac{\delta E}{h\omega_0}\right)$, then the solution is

$$\rho_{\delta E} \propto e^{-\frac{\delta E^2}{2\sigma_E^2}}, \quad \rho_\phi \propto e^{\frac{\cos(\delta\phi)}{\sigma_\phi^2}}$$

where $\delta\phi = \phi - \pi$ and $\sigma_\phi = \sqrt{\frac{2\pi h|\eta|}{E_0 e\hat{V}}}\sigma_E$.

A monte carlo is used to generate the above gaussian energy distribution and almost-gaussian phase distribution, with the distribution truncated at the separatrix boundary. A polynomial fit is done to calculate Δp as a function of Δt. <u>In this case Δ refers to the rms value</u>. The fit is shown in Figure 1. For $e\hat{V}$ = 1.05 MV at 150 GeV the result is

$$\Delta p = 34.9680 \bullet \Delta t - 6.4322 \bullet \Delta t^2 + 1.3028 \bullet \Delta t^3 - 0.1520 \bullet \Delta t^4$$

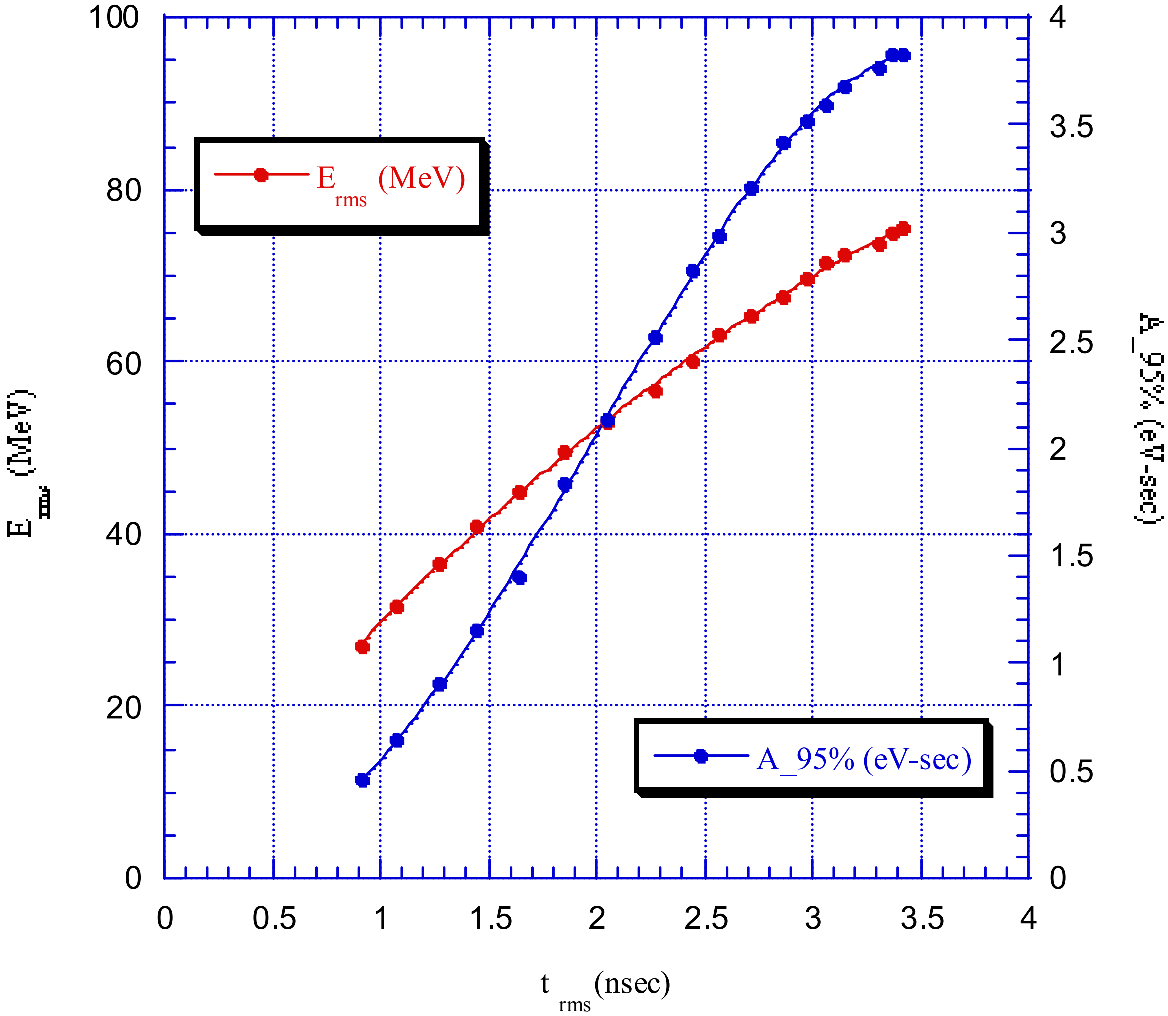

Figure 1: Polynomial fits for momentum spread and longitudinal emittance

Linear least square fits for Δt and Δp/p as functions of time are shown in Figure 2. Also shown are the SBD calculated values of Δp/p and the FW calculated values of Δp/p. RMS percent deviations from the linear fits are shown in Table 1.

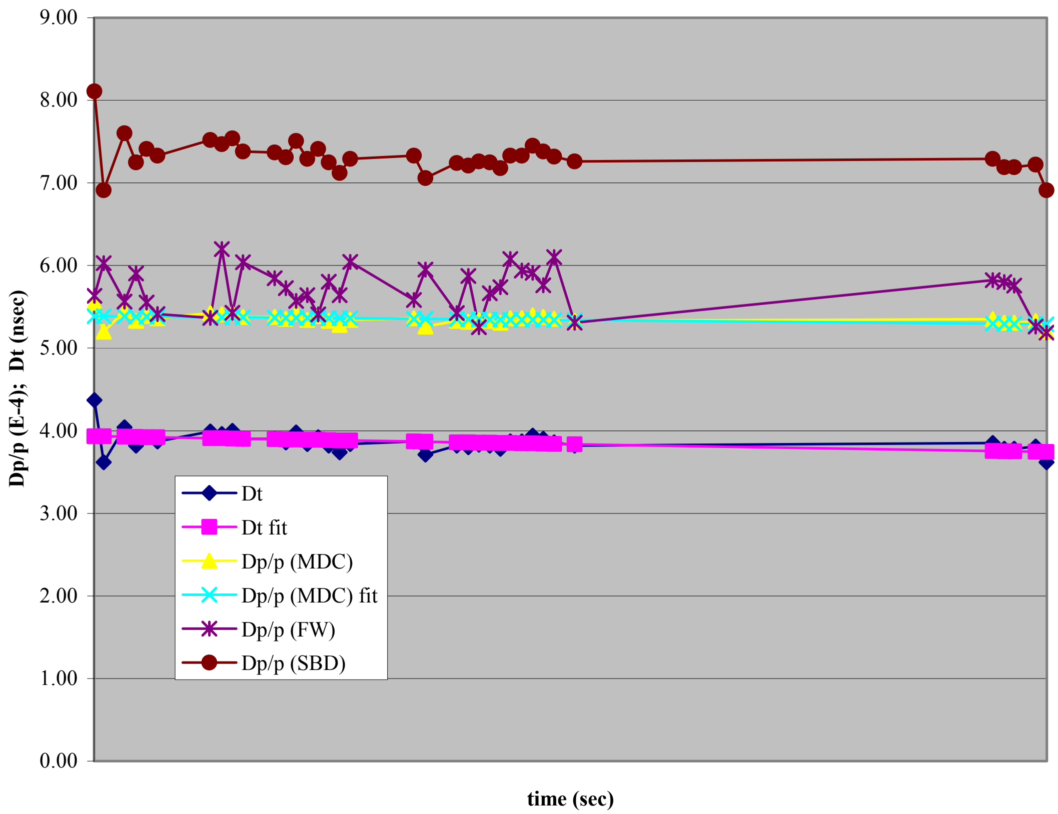

Figure 2: Δt and Δp/p and fits. Fits are not shown for the Δp/p values calculated by the SBD and FW

	RMS deviation	RMS % deviation
SBD Δt	.112 nsec	2.9%
MDC Δp/p	.057	1.1%
SBD Δp/p	.180	2.5%
FW Δp/p	.261	4.6%

Table 1: RMS deviations from straight line fit for Δt and Δp/p

<u>FW sigmas:</u>

FW sigmas (average of 1st and 2nd passes) are shown in Fig. 3. Linear least square fits to a straight line vs. time are also shown. The rms deviations and percent deviations from the linear fits are shown in Table 2.

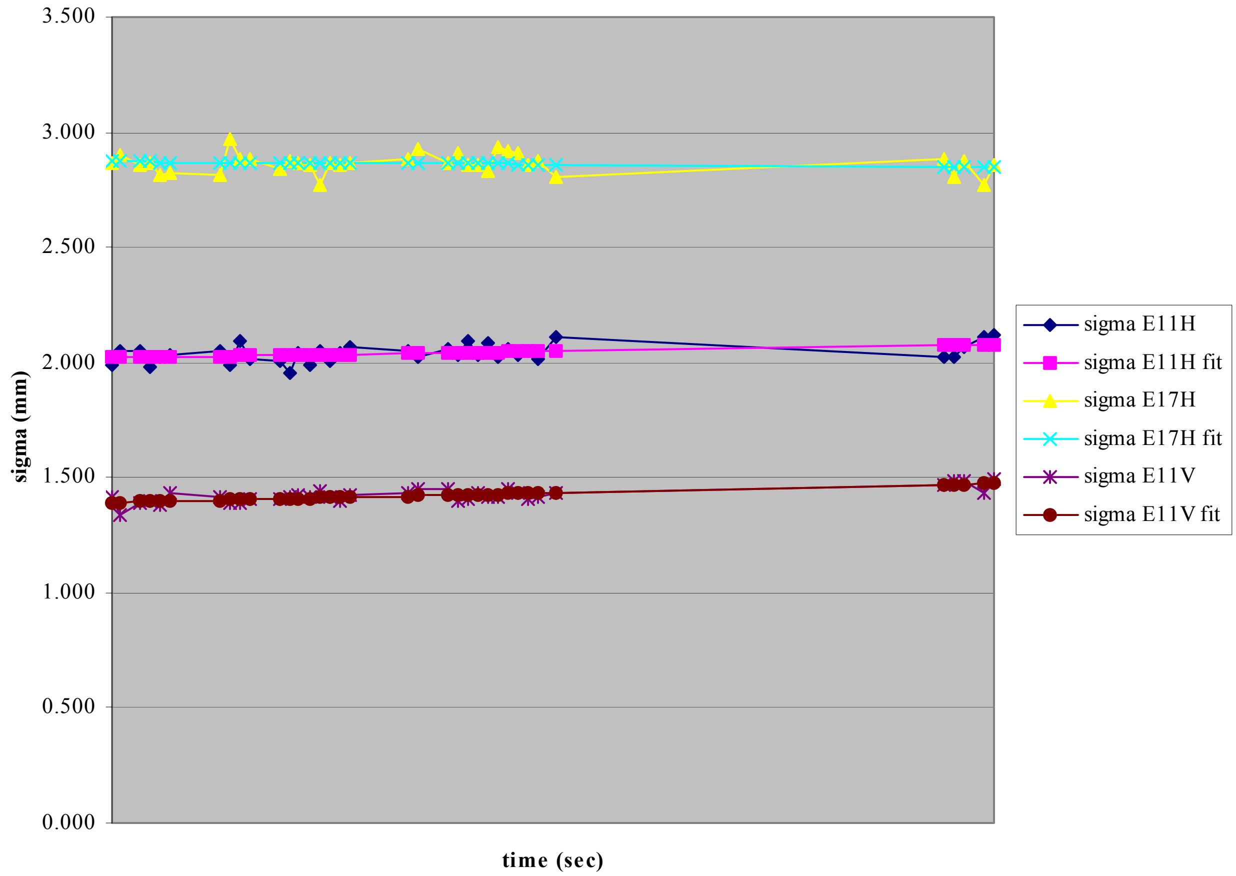

Figure 3: FW sigmas and fits. Data points are the average of 1st and 2nd passes.

	RMS deviation (μm)	RMS % deviation
E11H	34	1.7%
E17H	43	1.5%
E11V	21	1.4%

Table 2: RMS deviations from straight line fit for FW sigmas.

Transverse emittance:

The emittance is calculated with the formula

$$\varepsilon_i = \left(\sigma_i^2 - \left(\frac{\Delta p}{p} \right)^2 D^2 \right) \frac{6\gamma}{\beta} \qquad \text{[eq. 1]}$$

where i = E11H, E17H, or E11V; $\Delta p/p$ is from the fit from Figure 1; γ is the relativistic gamma; β and D are the lattice functions shown in Table 3.

	β (m)	D (m)
E11H	83.5	2.83
E17H	62.5	4.58
E11V	84.5	-0.09

Table 3: Lattice functions at FW @ 150 GeV as determined by V. Lebedev

Figure 4 shows the FW horizontal emittance calculated with eq. 1. The emittance measured by the E17 wire should be the same as the emittance measured by the E11 wire. Possibly the formula is incorrect – it is valid if both transverse and momentum distributions are ~gaussian, and it is known that the momentum distribution at 150 GeV is not gaussian. Increasing the E17 β by 25% and decreasing the E11 β by 25% gives passable agreement (see Figure 5), but it is generally agreed that the Tevatron β functions are known to better than 10% at 150 GeV [P. Bagley, V. Lebedev]. However, increasing the E17 dispersion function by 5% and decreasing the E11 dispersion function by 5% gives very good agreement between the two emittances (Figure 6). The final horizontal emittance is calculated from the weighted sum of the E11 and E17 emittances (with unadjusted lattice functions). The weights are the inverse squares of the rms deviations from the straight line fits. The relative E11/E17 weights are .86/.14, so, in effect, the E17 wire is hardly used at all. The final horizontal emittance and the FW front-end calculated emittance are plotted in Figure 7. The vertical emittance (calculated with eq. 1) and the FW front-end calculated vertical emittance is plotted in Figure 8. Note that the MI reported emittances at 150 GeV for P1 was 21.2 horizontal (bad!) and 26.6 vertical (very bad!), and the MI→Tevatron transfer efficiency was 89% (FBI narrow gate signals). Note also that if $\Delta p/p$ from the SBD is used in eq. 1, then negative numbers are obtained for the horizontal emittances. Table 4 shows the rms deviations and rms % deviations for all these emittances.

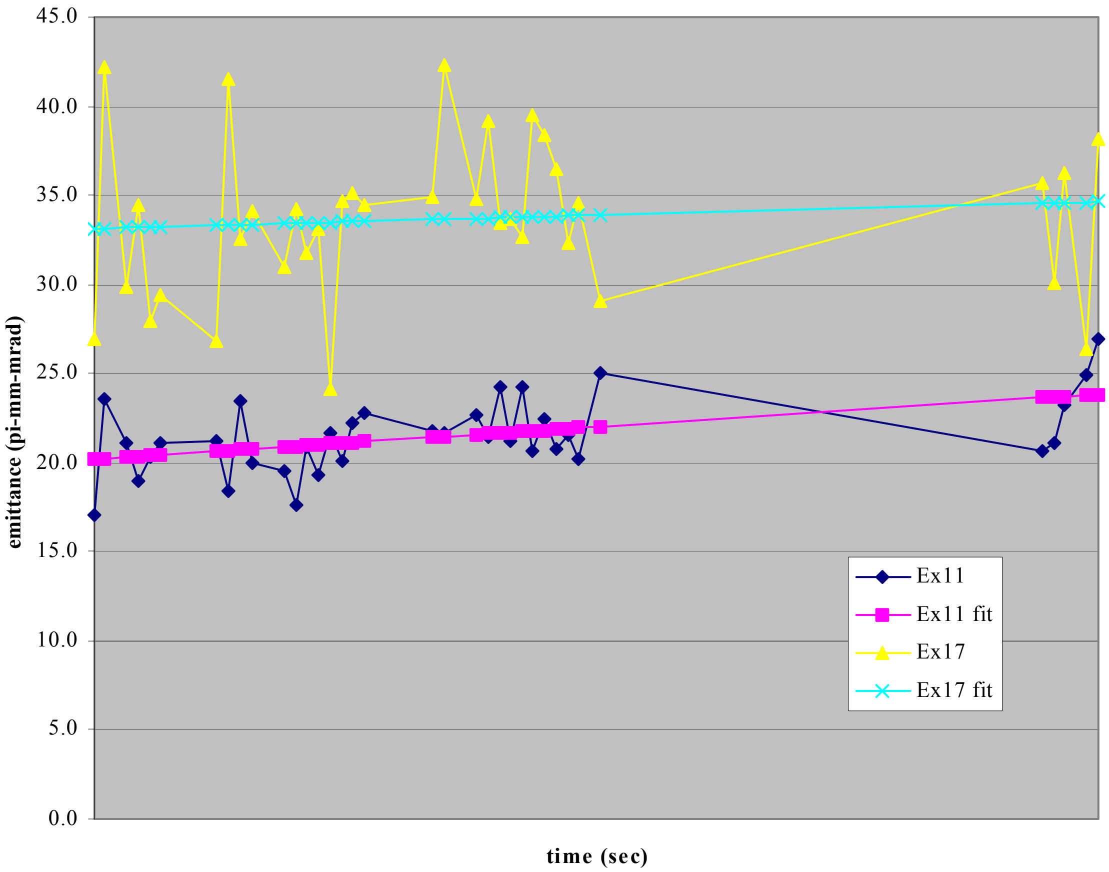

Figure 4: FW horizontal emittances and fits with original lattice functions

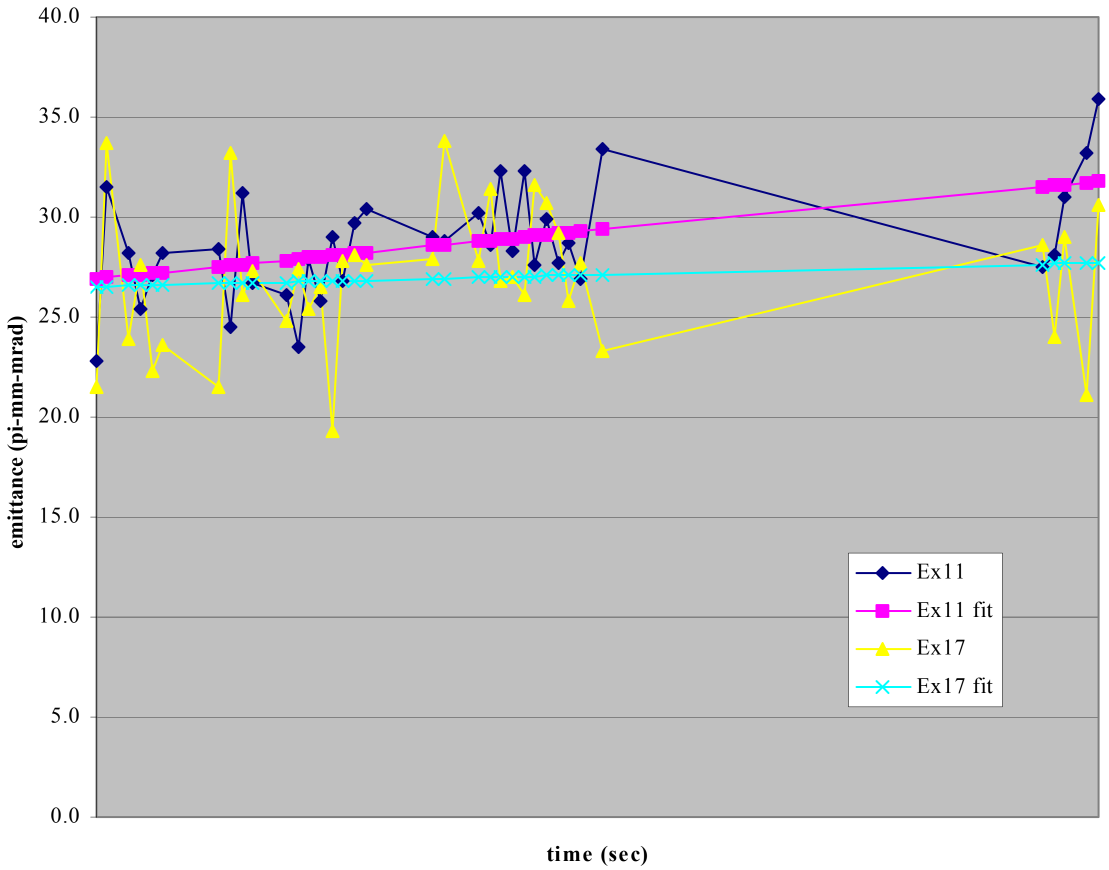

Figure 5: FW horizontal emittances and fits with β functions changed by 25%

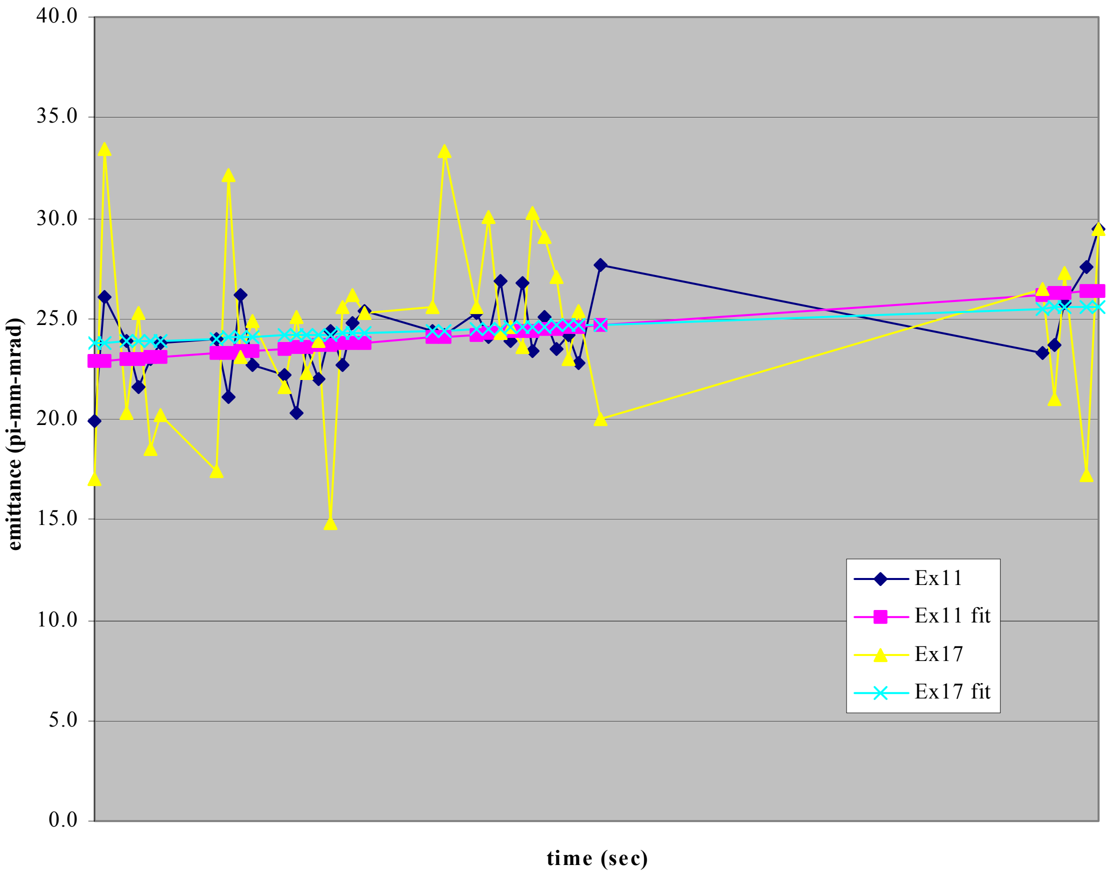

Figure 6: FW horizontal emittances and fits with dispersion functions changed by 5%

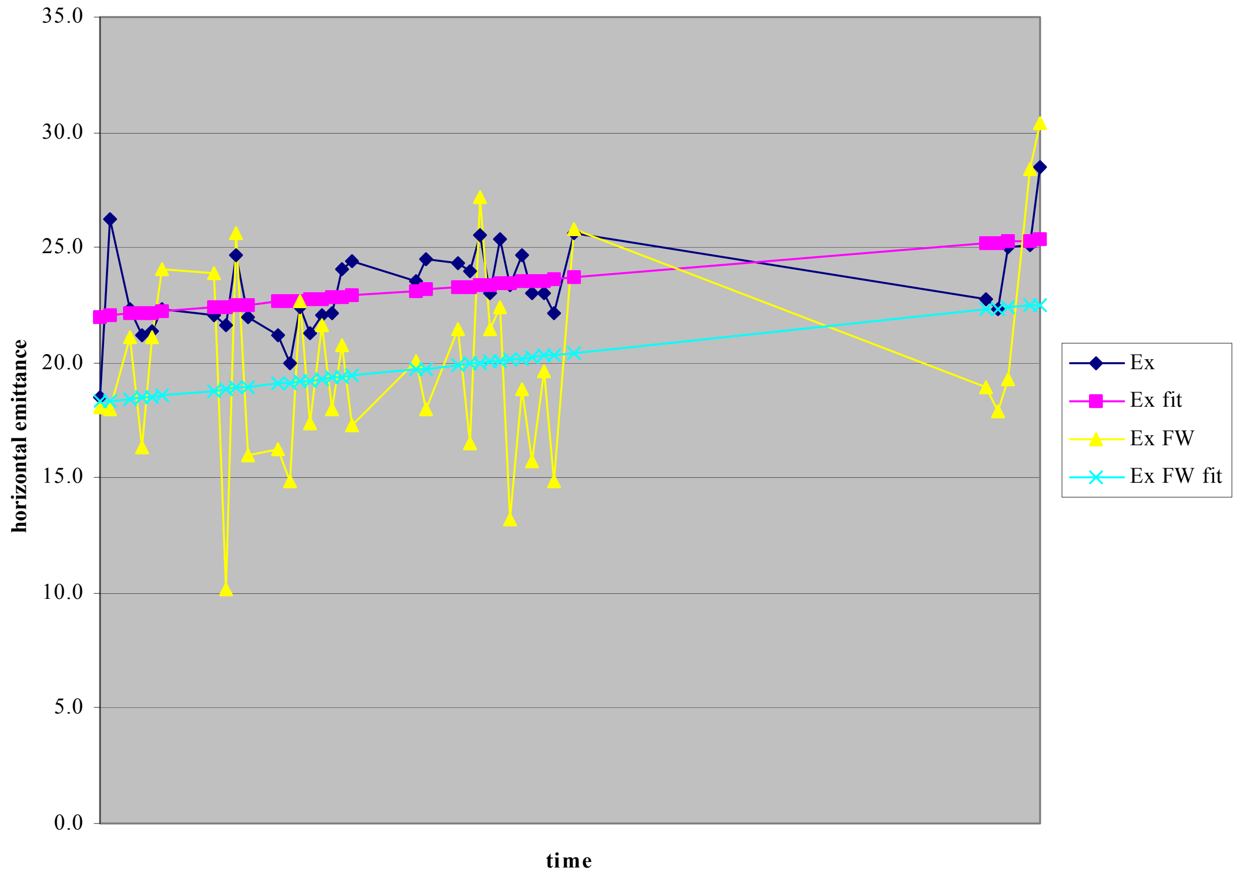

Figure 7: MDC calculated and FW front-end calculated horizontal emittances and fits

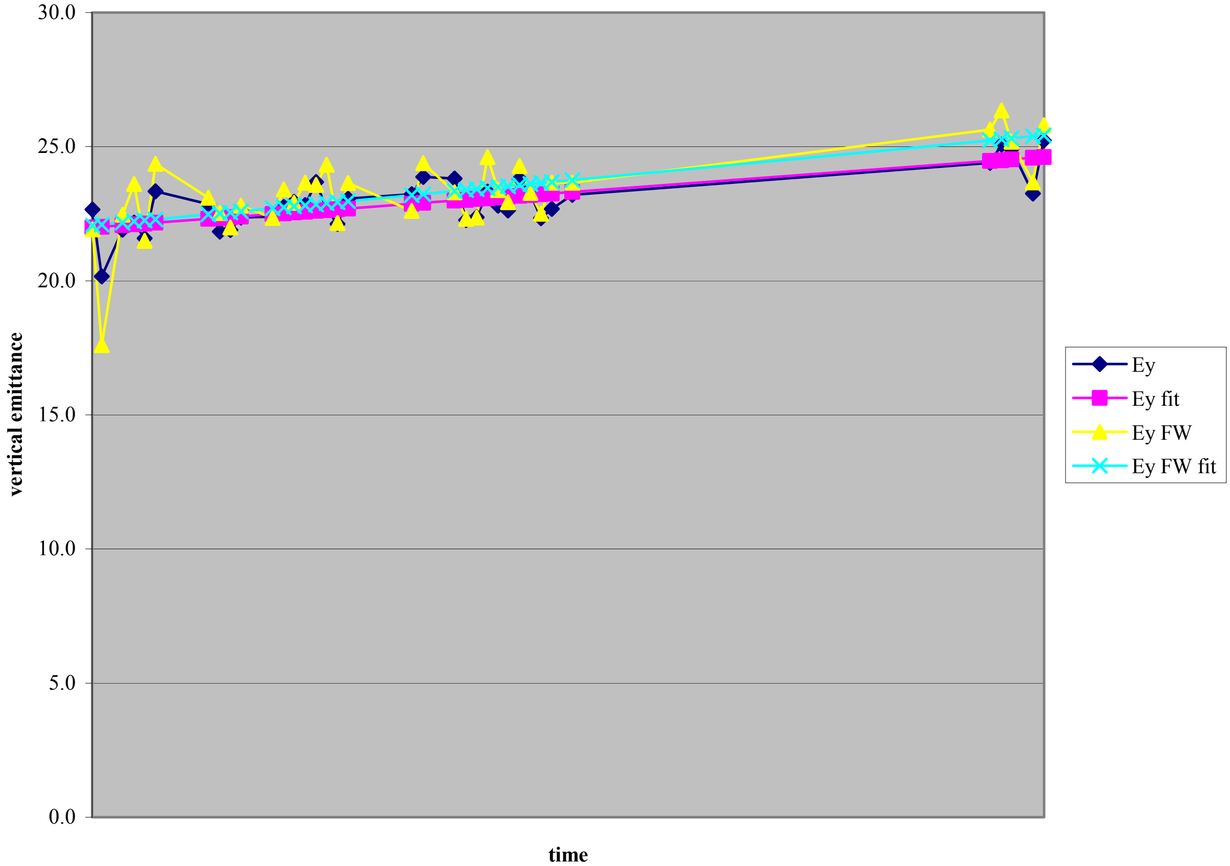

Figure 8: MDC calculated and FW front-end calculated vertical emittances and fits

	rms deviation (π-mm-mrad)	% rms deviation	Average emittance (π-mm-mrad)
$\varepsilon 11_H$ [MDC]	1.8	8.3%	21.5
$\varepsilon 17_H$ [MDC]	4.4	13.0%	33.7
ε_H [MDC]	1.6	7.1%	23.2
ε_H [FW]	4.1	20.7%	19.8
ε_V [MDC]	0.7	2.9%	22.9
ε_V [FW]	1.1	4.8%	23.3

Table 4: RMS deviations and % deviations from straight line fits for emittances

<u>Comments/conclusions:</u>

1) Emittance calculation using $\Delta p/p$ is more accurate than current method used in the FW front-end.
2) I will repeat this analysis at 980 GeV.

3) SBD front-end calculation of Δp/p at 150 GeV should be corrected. I will supply correct equation for 980 GeV (FT and LB).

4) FW front-end should read Δp/p from SBD (how?) and do the emittance calculation by the method demonstrated in this note. The Tevatron OAC should also do the calculation this way. SDA derived tables should also do the calculation this way.

5) I am not convinced there is a substantial, new emittance blow-up on the ramp. A more careful FW analysis is required to determine this. (After-note: V. Shiltsev contends that vertical emittance measurement is good enough to determine this.)

6) The proton transverse emittances in the MI @ 150 GeV for store 2070 are very large. Is this the norm?

7) This analysis does not show any evidence that any FW hardware is "broken".

8) This problem (Δp/p screwing up the emittance measurement) is worse now than in Run I because a) the longitudinal emittance is larger, and b) the dispersion at the E17 FW is larger.(?)

9) This analysis might be improved if the FW goodness-of-fit parameters were used as weights in the fitting procedures.